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Anisotropic, Spatially Homogeneous, Bulk Viscous Cosmological Model

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An anisotropic, spatially homogeneous, bulk viscous model of the universe without shear viscosity is investigated. The exact solutions of the model are derived and some physical consequences of the model are discussed.

1. INTRODUCTION

In order to study the evolution of the universe, many workers have investigated cosmological models with a fluid containing viscosities. Murphy (1973) obtained an exactly soluble cosmological model of the zero-curvature Friedman model in the presence of bulk viscosity alone. These solutions have the interesting feature that the big-bang singularity appears in the infinite past. Recently Mohanty and Pradhan (1989) studied the problem of Murphy (1973) for nonzero curvature of the Friedman model and derived solutions for their model. In the same paper they interpreted their result to explain the present status of the universe. Belinskii and Khalatnikov (1976), investigating a Bianchi type I cosmological model under the influence of viscosity, found the important property that near the initial singularity the gravitational field creates matter.

Szydlowski and Heller (1983) have constructed world models filled with interacting matter and radiation including bulk viscosity dissipation. They have shown the existence of stationary solutions in which the bulk viscosity term can be interpreted as phenomenologically describing the creation of matter and radiation. Santos *et al.* (1985) obtained exact solutions of an isotropic homogeneous cosmology with general viscosity for open, closed, and flat universes. Banerjee and Santos (1983, 1984) obtained some

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exact solutions for a homogeneous anisotropic model using certain restrictions. Banerjee *et al.* (1985) obtained some Bianchi type I solutions for the case of stiff matter by using the assumption that shear viscosity coefficients are power functions of the energy density. However, the bulk viscosity coefficients in the model are zero or constant. Recently, Huang (1988) presented exact solutions of a Bianchi type I cosmological model with bulk viscosity without introducing shear viscosity. However, he adopted the restriction that the viscous coefficients are constant or proportional to the energy density. Finally, Huang (1988) studied various physical aspects of the problem.

In this paper, we investigate an anisotropic, spatially homogeneous, bulk viscous model without introducing shear viscosity when the spacetime is described by a Bertotti–Robinson-type metric

$$ds^2 = a^2[n\,dx^2 + \tfrac{1}{2}n^2\,dy^2 + m\,dz^2 - (dt + n\,dy)^2] \tag{1}$$

where $a = \text{const}$, $m = m(x)$, and $n = n(t)$. Our model is quite different from that of Banarjee and Santos (1983, 1984). In Section 2 the field equations and their solutions are obtained and Section 3 gives concluding remarks.

2. FIELD EQUATIONS, CONSEQUENCES, AND THEIR SOLUTIONS

The relativistic field equation for a bulk viscous perfect fluid distribution may be written as

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -\frac{k}{4\pi} T_{ij} \tag{2}$$

and

$$U^i U_i = -1 \tag{3}$$

where Λ is the cosmological constant and the energy-momentum tensor for such a distribution is given by

$$T_{ij} = (\varepsilon + \bar{p}) U_i U_j + \bar{p} g_{ij} \tag{4}$$

with

$$\bar{p} = p + \eta U^i_{;i} \tag{5}$$

Here ε, p, η, and U_i correspond to energy density, internal pressure, bulk viscous coefficient, and four-velocity flow vector of the distribution, respectively, and a semicolon denotes covariant differentiation. The units are so chosen that the velocity of light $c = 1$ and the gravitational constant $G = 1$.

The explicit form of the field equations (2) and (3) for the metric (1) can be given as

$$G_{11} \equiv \frac{\ddot{n}}{n} m - \Lambda a^2 m = \frac{k}{4\pi}[(\varepsilon + \bar{p}) U_1^2 + \bar{p} a^2 m] \tag{6}$$

$$G_{22} \equiv \frac{n^2}{4m}\left(\frac{m''}{m} - \frac{m'^2}{m^2}\right) - \Lambda \frac{a^2 n^2}{2}$$
$$= -\frac{k}{4\pi}\left[(\varepsilon + \bar{p}) U_2^2 - \frac{\bar{p} a^2 n^2}{2}\right] \tag{7}$$

$$G_{33} \equiv \frac{\ddot{n}}{n} m - \Lambda a^2 m = \frac{k}{4\pi}[(\varepsilon + \bar{p}) U_3^2 + \bar{p} a^2 m] \tag{8}$$

$$G_{44} \equiv \frac{1}{2m}\left(\frac{m''}{m} - \frac{m'^2}{m^2}\right) - \Lambda a^2 = -\frac{k}{4\pi}[(\varepsilon + \bar{p}) U_4^2 - \bar{p} a^2] \tag{9}$$

$$G_{24} \equiv \frac{n}{2m}\left(\frac{m''}{m} - \frac{m'^2}{m^2}\right) - \Lambda a^2 n$$
$$= -\frac{k}{4\pi}[(\varepsilon + \bar{p}) U_2 U_4 - \bar{p} a^2 n] \tag{10}$$

$$G_{12} = G_{21} \equiv (\varepsilon + \bar{p}) U_1 U_2 = 0 \tag{11}$$

$$G_{13} = G_{31} \equiv (\varepsilon + \bar{p}) U_1 U_3 = 0 \tag{12}$$

$$G_{14} = G_{41} \equiv (\varepsilon + \bar{p}) U_1 U_4 = 0 \tag{13}$$

$$G_{23} = G_{32} \equiv (\varepsilon + \bar{p}) U_2 U_3 = 0 \tag{14}$$

$$G_{34} = G_{43} \equiv (\varepsilon + \bar{p}) U_3 U_4 = 0 \tag{15}$$

and

$$\frac{1}{2a^2 m}(U_1^2 + U_3^2) + \left(\frac{U_2}{an} - \frac{U_4}{a}\right)^2 = \frac{1}{2}\left[\left(\frac{U_4}{a}\right)^2 - 1\right] \tag{16}$$

Henceforth a prime and dot represent exact differentiation with respect to x and t, respectively.

As $U_4 \neq 0$ for a bulk viscous perfect fluid distribution, equations (13) and (15) yield

$$U_1 = 0 = U_3 \tag{17}$$

Thus, equations (11)–(15) are identically satisfied. Now comparing equations (9) and (10), we get

$$U_2 = U_4 n \tag{18}$$

with the help of which equations (16) and (18) yield

$$U_2 = an \tag{19}$$

and

$$U_4 = a \tag{20}$$

Now with the help of equation (17), the field equations (6) and (8) reduce to a single equation of the form

$$\frac{\ddot{n}}{n} - \Lambda a^2 = \frac{k}{4\pi} \bar{p} a^2 \tag{21}$$

with the help of equations (19) and (20), equation (7) reduces to

$$\frac{1}{2m}\left(\frac{m''}{m} - \frac{m'^2}{m^2}\right) - \Lambda a^2 = -\frac{k}{4\pi}[(2\varepsilon + \bar{p})a^2] \tag{22}$$

and equations (9) and (10) reduce to a single equation of the form

$$\frac{1}{2m}\left(\frac{m''}{m} - \frac{m'^2}{m^2}\right) - \Lambda a^2 = -\frac{k}{4\pi}\varepsilon a^2 \tag{23}$$

Equations (22) and (23) imply that

$$\bar{p} + \varepsilon = 0 \tag{24}$$

Again use of equation (24) in equations (22) and (23) yields a single field equation of the form

$$\frac{1}{2m}\left(\frac{m''}{m} - \frac{m'^2}{m^2}\right) - \Lambda a^2 = \frac{k}{4\pi}\bar{p} a^2 \tag{25}$$

It is evident from equations (21) and (25) that the situation corresponds to a physically realistic one if

$$\bar{p} = \text{a negative constant} \tag{26}$$

Thus, from equation (24) we conclude that the energy density ε is positive and is equal to a constant throughout the distribution.

Now for the metric (1), equation (5) takes the following form:

$$\bar{p} = p - \eta \frac{\dot{n}}{an} \tag{27}$$

The Bianchi identity, i.e., $T^i_{j;i} = 0$, leads to the following equations:

$$\frac{\bar{p}_1}{m} = \frac{\bar{p} m'}{2m^2} \tag{28}$$

$$\bar{p}_4 = 0 \tag{29}$$

The subscripts 1 and 4 after the field variables represent partial differentiation with respect to x and t, respectively.

It is evident from equations (26), (28), and (29) that

$$m = \text{const} \tag{30}$$

Again using (30) in equation (25), we get

$$\frac{k}{4\pi}\bar{p} + \Lambda = 0 \tag{31}$$

Now with the help of equation (31), equation (21) reduces to

$$\ddot{n} = 0 \tag{32}$$

which yields

$$n = At + B \tag{33}$$

where A and B are arbitrary constants.

Using (31) and (33) in equation (27), we get

$$p = -\frac{4\pi}{k}\Lambda + \eta\frac{A}{a(At+B)} \tag{34}$$

Now we consider the following particular cases in order to study the explicit physical behavior of the model:

(i) $\eta = \eta_0$ ($=$const).
(ii) $\eta = p$.

Case (i). $\eta = \eta_0$ ($=$const). In this case we find that as $t \to 0$, $p \to (A\eta_0/aB - 4\pi\Lambda/k)$ and $n \to B$, and as $t \to \infty$, $p \to -4\pi\Lambda/k$ and $n \to \infty$. Thus, we can predict that at an initial epoch the universe was flat and homogeneous and at the infinite future there will be a big crunch even though matter is strictly homogeneous. This indicates that the universe started evolving from an initial isotropic situation to that of anisotropy.

Case (ii). $\eta = p$. In this case we find that the bulk viscous coefficient

$$\eta\ (=p) = \frac{-(4\pi/k)\Lambda}{1 - A/(At+B)a}$$

Here we find that as $t \to 0$, $\eta\ (=p) \to -(4\pi\Lambda/k)/(1 - A/Ba)$ and $n \to B$, and as $t \to \infty$, $\eta\ (=p) \to -4\pi\Lambda/k$ and $n \to \infty$.

Since $\eta > 0$, the model corresponds to a physically realistic situation when the cosmic time $t < (1/a - B/A)$ for $\Lambda > 0$ and $t > (1/a - B/A)$ for $\Lambda < 0$, assuming $a > 0$. The situation is interchanged if $A < 0$. In this case the physical behavior with regard to the pressure of the distribution and the metric potential is nearly similar to that of the preceding case.

3. DISCUSSION

One can infer from equation (34) that the ratio η/p increases with the age of the universe. This indicates that when the universe is filled with liquid the temperature and pressure of the universe decreases as η increases during evolution. Thus, the interparticle distance increases, indicating the expansion of the universe and the motion of the liquid gradually becomes streamlined with an increase of η. The other possibilities pertaining to equation (34) lead to various peculiar physical situations which need further study to reveal the physics involved in the problem.

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